

$$\frac{1}{2}(\rho_{az}^L + \rho_{az}^V) = A'' + B''T \quad (6)$$

are given in Table 2. The magnitude of the correlation coefficient for the temperature-density relationship is similar to that for the composition-density relationship. (A value of the correlation coefficient close to  $\pm 1$  indicates a high correlation between two variables and a value near 0 indicates low correlation. It is thus a measure of the "goodness of fit".) Once again, the data for ethanol-benzene azeotropes (when the azeotropic locus does not end at a critical azeotropic point for mole fractions between 0 and 1) can also be fitted to Equation (6), although there are insufficient data points to reach a definite conclusion.

In conclusion, the rectilinearity rule proposed by Cailletet and Mathias can be applied to saturation densities along the azeotropic locus, both as a function of temperature and of composition. The rule may have further applications to other properties of binary mixtures.

#### NOTATION

A, B, C = constants  
P = pressure  
T = temperature, K  
x = composition  
 $\rho$  = density, kmol m<sup>-3</sup>

#### Superscripts

L = liquid  
V = vapor  
c = critical

#### Subscripts

i = component i  
az = azeotrope

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## Measurements of Turbulent Flow Velocity for Sudden Expansion Cylindrical Tube Using Laser Doppler Velocimeter (LDV)

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A laser Doppler velocimeter (LDV) is an apparatus for measuring frequency change of the scattered light (Doppler shift) from a particle moving inside a fluid. The Doppler shift is used to measure the velocity and the particle velocity is then used to infer the fluid velocity. A hot wire velocimeter has long been used for measuring a turbulent flow velocity, but the instrument has a drawback, in that it produces a disturbance inside the flow field. An LDV does not have the probe disturbance problem, because only light beams are passing through the flow field. Discussions of the LDV techniques can be found in the cited references.

In this study, an argon ion laser was used as a light source to measure the turbulent air flow velocity along the center line and radially, at certain cross sectional areas of a sudden expansion cylindrical tube. Results show that the center line velocity de-

creases to about 25% of the inlet velocity at a distance of about eight inlet diameters downstream ( $X/D = 8$ ). A negative cross sectional velocity profile is observed at a distance of  $X/D = 1.25$  from the tube entrance, which indicates that a circulation flow exists before the boundary layer reattachment.

#### EXPERIMENTAL APPARATUS

##### Flow System

A circular plastic tube 0.56m long, 101.6mm ID, and about 6.35mm thick was used as a test section. About 25.4mm thick and 152.4mm diameter circular plates, with a 50.8mm diameter hole at the middle of the plates, were attached co-axially to both ends of the 0.56m long tube. The back end of the tube was connected to a vacuum cleaner, and the air was sucked in through the 50.8mm diameter hole and expanded instantaneously to the 101.6mm ID plastic tube, (Figure 1). Baking soda

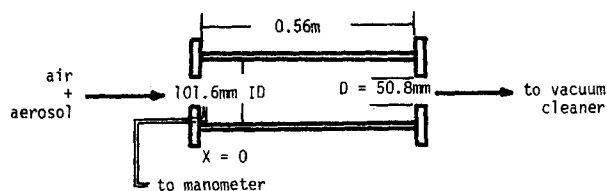


Figure 1. Flow system.

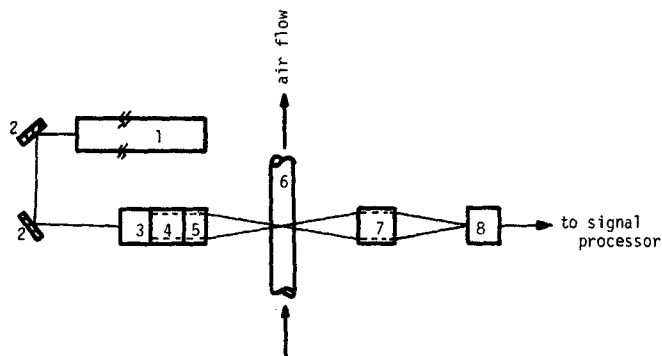


Figure 2. Optical system

- |                      |                    |
|----------------------|--------------------|
| 1. Argon ion laser   | 5. Focusing lens   |
| 2. Refraction mirror | 6. Circular tube   |
| 3. Beam splitter     | 7. Collecting lens |
| 4. Frequency shifter | 8. Photo detector  |

solution consisting of an average diameter of  $0.5\mu$  aerosol particles was sprayed into the front end of the circular tube. The particles were used as the seeds in the flow field to scatter the focused light. A small pitot tube was used to measure the inlet air velocity.

### Optical System

In general, an LDV system is made up of two parts, an optical system and a signal processor. In the optical system, a laser beam is separated into two parallel beams of equal intensity. These beams are then focused at a point where the particles pass through and light scatters. The scattered light is collected by a photodetector and converted to an electrical signal. The signal processing units are then used to convert the frequency of the electrical signal to a voltage signal, and the voltage signal is used to calculate the particle velocity. The optical system used in this study is depicted in Figure 2. In this study, the circular tube is fixed to the ground and the entire optical system is mounted on a movable table. The table can be adjusted accurately in axial and radial directions, so that the laser beams can be focused at any positions inside the circular tube for which the flow velocity can be measured.

### EXPERIMENTAL RESULTS

The velocity profile along the center line is shown in Figure 3. In the beginning, the velocity decreases slowly between the inlet and a downstream distance of  $X/D = 2$ . It then decreases rapidly and reaches about 25% of the inlet velocity at a downstream distance of  $X/D = 8$ . At this point, the velocity profile becomes a fully developed flow. This result agrees with that discussed by Moon and Rudinger (1977). They showed that the fully developed flow occurs at a distance of about four duct diameters from the entrance, which is equivalent to  $X/D = 8$  in this study. The cross sectional velocity profiles at  $X/D$  equal to 1.25, 3.75, 6.25 and 9.75 are shown in Figure 4. Negative velocity shown at the cross section of  $X/D = 1.25$  indicates that the boundary layer separation zone exists between the wall and the radial distance of  $r/R = 0.68$ . Inside this separation region, the particles move in the opposite direction of the main flow. The boundary layer reattachment for a sudden expansion flow should occur between a cross section of  $X/D = 1.25$  and 3.75 according to this study. A plug-shaped velocity profile for the fully developed turbulent flow is seen at  $X/D = 9.75$ .

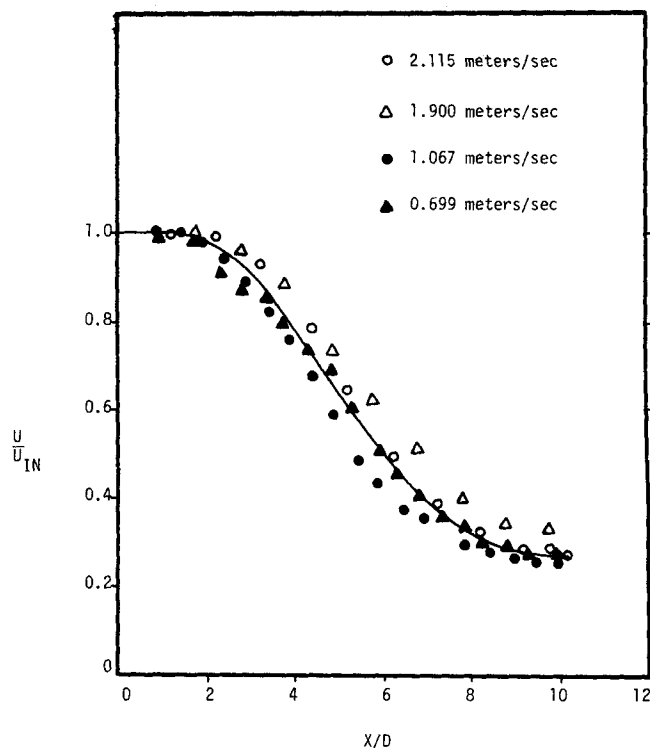


Figure 3. Velocity profile at center line.

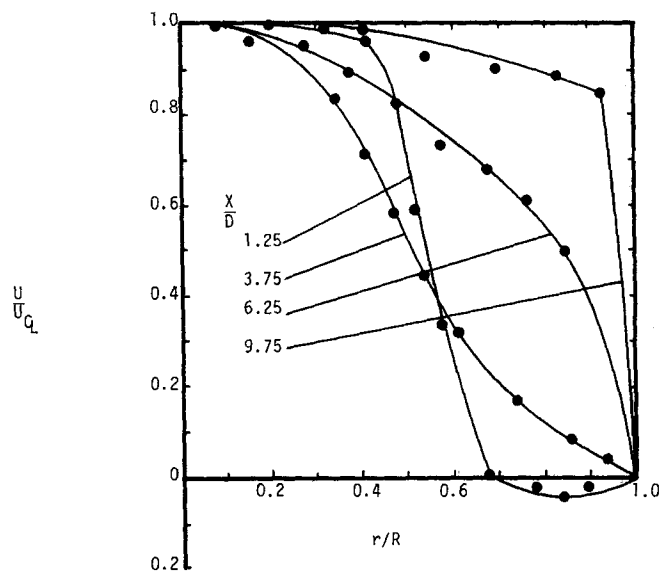


Figure 4. Velocity profiles for cross sections.

### ACKNOWLEDGMENT

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### NOTATION

- |          |  |
|----------|--|
| $D$      | = inlet diameter of testing tube, 50.8mm |
| $r$      | = radial distance of testing tube        |
| $R$      | = radius of testing tube, 50.8mm         |
| $U$      | = local velocity                         |
| $U_{CL}$ | = center line velocity                   |
| $U_{IN}$ | = air inlet velocity                     |
| $X$      | = axial distance from inlet point        |

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# Process Identification of Open-Loop Unstable Systems

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Frequency response information plays a vital role in the design of process control systems. This information can be developed from a dynamic mathematical model or from experimental tests on the plant itself. Among the experimental techniques, step testing and pulse testing are the most widely used for dynamic identification. A review of literature revealed many applications of the pulse and step testing method for identification of the open-loop stable processes (for example, see Clements and Schnelle 1963, Hogen 1964, Schork and Deshpande 1978). But there appears to be no published information on experimental methods for identifying open-loop unstable processes. Instead, investigators have relied on dynamic mathematical models to obtain the open-loop transfer functions (for example, see Hopkins 1976). Of course, this is understandable, since experimental open-loop tests on such processes are not desirable.

An important example of an open-loop unstable system is an exothermic chemical reactor. In this case, a decreased heat transfer rate, accomplished by changing the rate of coolant flow, increases reactor temperature at which the reaction rate is higher which further increases the temperature. Thus, the system is open-loop unstable. The system can be stabilized by providing sufficient feedback by means of a controller. To determine suitable tuning constants for this controller, the open-loop transfer functions must be available. These tuning constants can be obtained if a dynamic model of the open-loop system is available. But in the absence of kinetic data, theoretical development of the model is not feasible.

The object of this work is to describe a novel technique for dynamic identification of open-loop unstable processes. The technique has been applied to a commercial exothermic chemical reactor which is used in the manufacture of a polymer. The method yields an approximate transfer function of the open-loop process, and is helpful in developing suitable tuning constants for the feedback controller.

## THE METHOD

The block diagram of a typical exothermic reactor control system is shown in Figure 1. The reactor is assumed to have a transfer function arising from a first-order lag plus dead-time model. The form chosen for the transfer function is typical for many polymerization systems.

Our objective is to identify the open-loop transfer function and also the parameters of the reactor model. We assume that

the dynamics of the measuring element and the final control element are known and can be expressed as

$$H(s) = \frac{k_T}{\tau_T s + 1} \quad (1)$$

where

$k_T$  = transmitter gain, psi/0<sub>c</sub>

$\tau_T$  = transmitter time constant, min.

and

$$G_r(s) = \frac{k_r}{\tau_r s + 1} \quad (2)$$

where

$k_r$  = valve gain cal/hr/psi

$\tau_r$  = valve time constant, min.

The closed-loop transfer function of the system of Figure 1 to changes in set point is given by

$$\frac{T(s)}{R(s)} = \frac{G_c(s) G_r(s) G_p(s)}{1 + G_c(s) G_r(s) G_p(s) H(s)} \quad (3)$$

For simplicity Equation (3) is written as

$$\frac{T(s)}{R(s)} = \frac{G(s)}{1 + G(s) H(s)} \quad (4)$$

where  $G(s)$  is the product of the transfer functions in the numerator of Equation (3).

The application of the present method requires a unity feedback on the closed-loop. Therefore, the block diagram of Figure

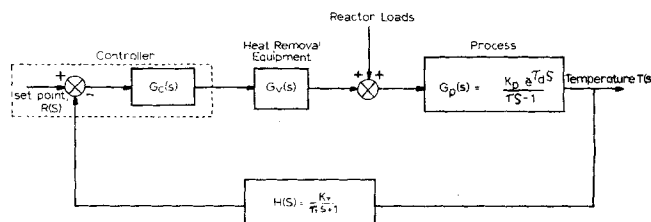


Figure 1. Block diagram of the reactor control system.

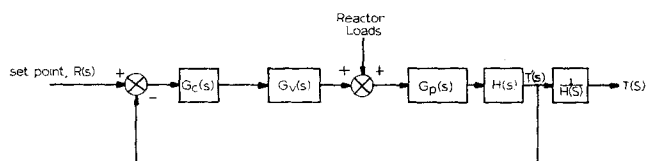


Figure 2. Block diagram of equivalent reactor control system having a unity feedback.